Production function and monopoly

$$Q(K;L) = K^2 L^{0.5}, \qquad K = 2$$
$$w = 8; \qquad r = 5$$

Assuming the inverse market demand as: $P = -2Q + 30$, obtain P^* and Q^* in Monopoly and Market Power (Abba Lerner Index).

Solution

Considering that the capital is fixed, the production function is:

$$Q = 4L^{0.5}$$

The firm's costs are:

$$C = 8L + 5 * 2 = 10 + 8L$$

Clearing L from the production function:

$$\frac{Q^2}{4^2} = \frac{Q^2}{16} = L$$

Inserting the value of L to obtain the cost function:

$$C(Q) = 10 + 8\frac{Q^2}{16} = 10 + \frac{Q^2}{2}$$

Therefore, the marginal cost function:

$$\frac{dC}{dQ} = MC = Q$$

On the other hand, the revenue is:

$$R = P * Q = (-2Q + 30)Q = -2Q^2 + 30Q$$

The marginal revenue:

$$MR = -4Q + 30$$

Equating marginal revenue to marginal cost:

$$Q = -4Q + 30$$

$$Q = 6$$

Then the monopolist's price will be:

$$P = -2 * 6 + 30 = 18$$

The market power:

$$\frac{P - MC}{P} = \frac{18 - 6}{18} = \frac{12}{18} = \frac{2}{3}$$